Statistics of the largest sunspot peaks

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Abstract

By considering the technique of Gumbel's first asymptotic distribution, the maximum solar activity for the next solar cycle is estimated. A maximum value not smaller than 68.4 and not greater than 187.4 monthly mean sunspots is predicted. Only a small number of sunspot predictions for solar cycle 24 peak are out of this interval. According to these values the solar cycle 24 could be smaller or higher than the prior one but not more than a 56% percent The interest to study the sun's cycles is based on the possibility to protect our environment, technological systems and human activity affected by solar aggressions. These effects are related to solar activity and geomagnetic field. Specially, magnetic disturbances induce electric currents in long conductors such as pipelines causing irregular pipeline corrosion.

Introduction

Many prediction methods of the magnitude and timing of Cycle 24 are used by Space Weather groups. These studies will help to estimate orbital drag an and other consequences of Space Weather in solar cycle 24.

Kane (2011) predicted the size of solar cycle 24 based on solar parameters during sunspot minimum and Kane 2007) also estimated the size of the solar cycle 24, based on Ohl's precursor method.

Pesnell (2007) in Predictions of solar cycle 24, NASA , published 45 predictions of de magnitude and timing of cycle 24, from http://www.swpc. noaa.gov/Solar Cycle/SC24/May_24 _2007_ table.pdf; and Pesnell (2008) also predictied maximum solar cycle 24,

Usoskin *et al*.(2009) studied the sudden systematic occurrence of sunspots at high solar latitudes in 1793–1796. They verified that a new cycle started in 1793, which was lost in the Wolf sunspot series. All events considered point out the necessity to review the sunspot series in the 18th century.

A great number of methods have been developed to predict the maximum sunspot number related to each solar cycle. Wilson (1990), and Silbergleit *et al*. (2001) used precursor techniques to deduce the cycle 22's and 23´s maximum amplitudes.

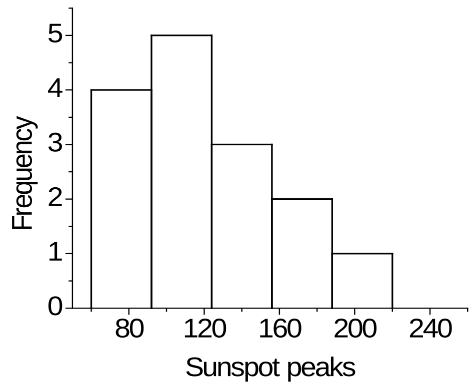

Figure 1. Histogram of the used monthly mean sunspot peaks.

The Gumbel's asymptotic distributions were applied by Siscoe (1976b) to study the largest geomagnetic storms in nine solar cycles. He applied the statistics of extremes to the first, second and third largest geomagnetic storms in nine solar cycles (viz. 11 to 19), as measured by the average half-daily aa index. Gao (1986), studied the time predictions of annual mean Ap index also applying the Gumbel's first distribution. Silbergleit (1996) also studied geomagnetic storms using the third Gumbel's distribution.

Willis *et al*. (1979) found that extreme areas of umbrae and whole spots have a diversion similar to that published by Siscoe (1976a) for the extreme values of sunspot number whereas the extreme areas of faculae have a smaller dispersion which is comparable to that found by Siscoe (1976b) for the largest geomagnetic storm per solar cycle.

Svalgaard *et al*. (2005) using polar field measurements, available for four solar cycles, predicted that the approaching solar cycle 24 will have a peak smoothed monthly sunspot number of 75 ± 8, making it potentially the smallest cycle in the last 100 years.

Duhau (2009) presented evidence that solar activity is in a declining episode that started about 1993. A value for maximum sunspot number in solar cycle 24 (87.5±23.5) is estimated.

D.Brown, E. Hupp, B. Steigerwald, and N. Neal-Jones. NASA Aids in Resolving Long Standing Solar Cycle

Mystery. RELEASE : 06-087, 2006 report available at http://mynasa.nasa.gov/home/hqnews/2006/mar/HQ_06087_solar_cycle.html predicted a new cycle with 30 to 50 percent stronger than the previous one.

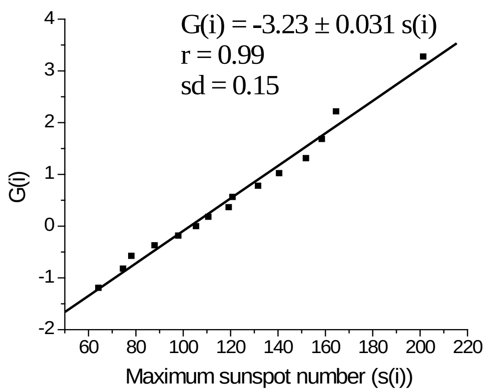

Figure 2. The data are plotted by using the ordinate values equal to *–ln[-ln (P i)]* and are indicated by squares. The P_i values are obtained by Eq. (5).

Silbergleit (2012) evaluated the possible solar peak for the current cycle.

The present article uses the statistics of extreme values to investigate the statistical properties of the largest sunspots per solar cycle and to predict solar cycle 24 peak.

Method

Often, it is not important to know the distribution of variables which describes a population. When we are interested to study the tails of the series, (the lowest or highest values) and the observed data appear to be normally distributed, we can consider the extreme value statistics.

Figure 1 shows the histogram of the data considered (downloaded at ftp:// ftp.ngdc.noaa.gov/STP/SOLAR_DATA/SUNSPOT_NUMBERS/MONTHLY and ftp://ftp.ngdc.noaa. gov /STP/SOLAR_DATA/SUNSPOT_NUMBERS/ maxmin.new). It indicates the nature of the observations (as pointed by Krumbein *et al.*[3]).

The tendency toward developing a tail to the right, characteristic of extreme distributions, then the Gumbel's first asymptotic distribution will be used (Gumbel, 1954).

For a given maximum observation, the probability that this value be less than s is defined as $p = \Phi(s)$. The probability that this value be equal or grater than s is $P = [1-\Phi(s)]$.

The theory of extremes gives the mathematical expression of Φ (s):

$$\Phi(s) = \exp\{-\exp[-(\alpha + \beta s)]\} \quad (1)$$

	Sunspots (s_i)	Plotting values (G_i)
1	64.2	-1.192660
2	74.6	-8.203667E-01
3	78.1	-5.743698E-01
4	87.9	-3.689795E-01
5	97.9	-1.812341E-01
6	105.4	-4.200988E-04
7	110.6	1.802902E-01
8	119.2	3.665129E-01
9	120.8	5.640188E-01
10	131.6	7.799466E-01
11	140.5	1.024586
12	151.8	1.315026
13	158.5	1.684694
14	164.5	2.217379
15	201.3	3.277027

Table 1. Maximum sunspots per solar cycles and the plotting values considered in the present article.

were α and β are constants determined by a linear square fit.

As the probability function for the maximum amplitude for each solar cycle is not known, the observed values of $\Phi(s)$ are deduced according to Gringorten (1963).

For *N* observed extreme values (where *N* is the size of the last fifteen sunspot cycles) the relationship between $\Phi(s)$ and *s* is obtained.

Extreme magnitudes are calculated by considering the solar cycles observed between the years 1830 and 2004 (see Figure 2) and by using the maximum sunspot numbers per cycle (s_i) in ascending order: $s_1 < s_2 < \ldots < s_{15}$ as it is shown in Table 1.

For each observed peak values it was assigned a probability according to:

$$P_i = (i - 0.44) / (N + 0.12) \quad (2)$$

where *i* is the ordinal number and *N* is equal to 15. The related G_i values are defined by:

$$G_i = -\ln[-\ln(P_i)] \quad (3)$$

The return periods T(s) and t(s) are calculated by using the two expressions:

$$T(s)=[1-\Phi(s)]^{-1}$$

$$t(s) = T(s)\ [T(s)\ -1]^{-1} \quad (4)$$

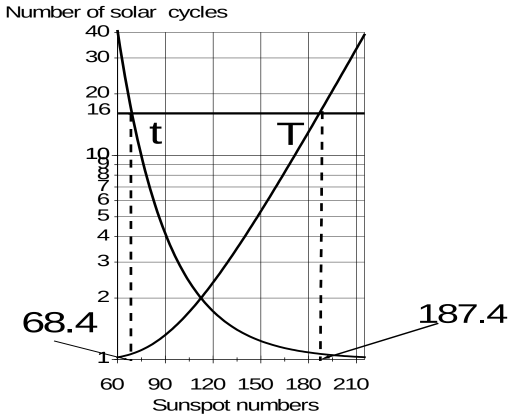

Figure 3. The ascending branch shows the waited number of periods required to detect one with extreme equal to or exceeding s. The descending one exhibits the expected number of intervals necessary to obtain one period with the extreme value less than s.

Eqs. (4) and (5) define *T(s)* and *t(s).* According to that, they are the expected times required to have one solar cycle maximum amplitude with the extreme equal to or exceeding and less than s.

The median value (*mv*), which is the mid-point of the distribution is calculated by plotting the observations calculated from Figure 3 for *T(s) = t(s) = 2,* the related abscissa is equal to *mv*. According Gumbel (1954), the arithmetic mean (*am*), which is the average of all results is related to the standard deviation *Sd* (as defined by [5]) by $Sd = 1.2825/\alpha$.

Results

The measure of the scatter as called Rd (relative dispersion), it is obtained to divide Sd by the mode. The statistical characteristic parameters of extreme values obtained are: the mode (*mo*), the median (*mv*) and the mean (*am*) equals to: 101.8, 116.2 and 120.5 sunspots respectively, with *Sd* = 41.4 sunspots and *Rd* = 0.41.

Figure 2 shows the best fitting adjustment obtained (chi square fit equal to 0.99), considering the Gumbel's first asymptotic distribution. The values of the parameters of Eq. (1) are: $\alpha = (-3.23 \pm 0.13)$ and $\beta = (0.031 \pm 0.001)$ sunspots.

The statistical characteristic parameters of extreme values obtained are: the mode, the median and the mean equals to: *mo* = 101.8 sunspots, *mv* = 116.2 and *am* = 120.5 sunspots respectively, with *Sd* = 41.4 sunspots and *Rd* = 0.41.

Conclusions

Figure 3 shows the return periods vs. sunspot numbers, the upper and lower bounds are shown for 16 solar cycles. In this case fourteen of them will be in the interval defined by 68.4 and 187.4 sunspots.

According Table 1, two peaks (numbered cycles 14 and 19) are out of the defined bounds, then the next sunspot peak will be between the mentioned extreme values. Following Gumbel's theory, a maximum value not smaller than 68.4 and not greater than 187.4 monthly mean sunspots is predicted. According to that, the solar cycle 24 could be smaller or higher than the prior one but not more than a 56% percent. The 95 percent of the maximum sunspot number in solar cycle 24, previously published, are included in the above mentioned interval (see: http://users.telenet.be/ j.janssens/SC24.htm).

Analysis of large sunspot numbers could be useful to diminish the effects of future solar aggressions on our environment, technological systems and human activity.

Space weather phenomena have different effects on technology. Energetic particles thrown out from the sun could produce magnetic disturbances and increased ionization in the ionosphere.The high energy particles affect satellites causing equipment damage on satellites. Radio waves used for communications or GPS are affected by the increased ionization with disruption of the communication or navigation systems. Magnetic disturbances induce electric currents in long conductors such as pipelines causing irregular pipeline corrosion.

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